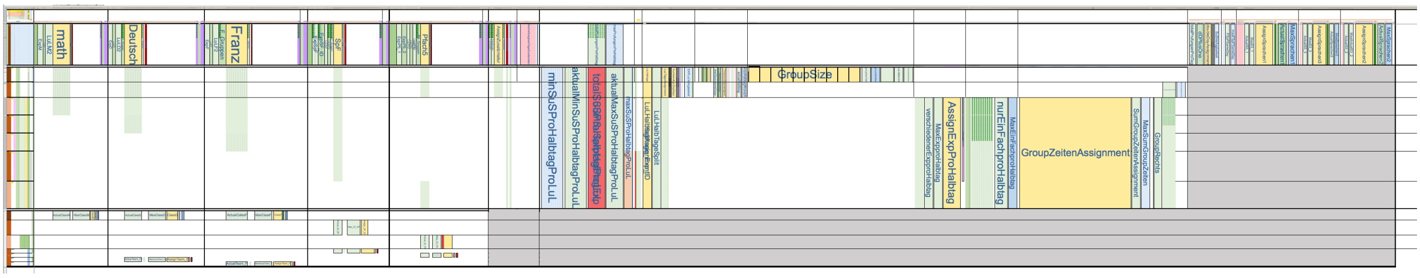
Constraint Satisfaction Problems:

From the front of the class, to the back of the school.

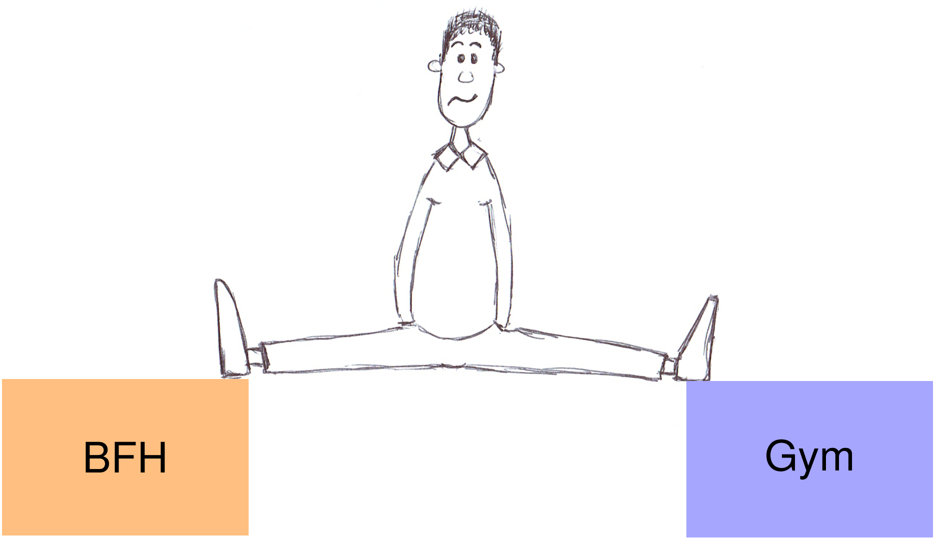


Geoff Ostrin

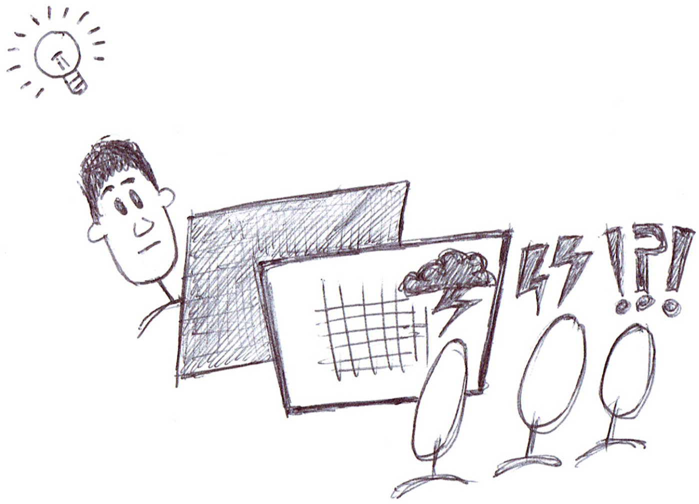
Berner Fachhochschule & Gymnasium Thun

Bern, 26.10.2019

What's the Story?



Serendipity



Constraint Satisfaction Problems:

From the front of the class, to the back of the school.

Constraint Satisfaction Problems

Find

$$X \text{ and } Y$$

Decision Variables

such that we maximise

$$Z = 300X + 500Y$$

Objective Function

while satisfying

$$\begin{aligned} X &\leq 4 \\ 3X + 2Y &\leq 18 \\ X &\geq 0 \\ Y &\geq 0 \end{aligned}$$

Constraints

Assignment Problems

	Email	Organise	Print	Arrange
Leonard	25	32	47	50
Michel	54	36	23	90
Matthias	85	23	52	53
Geoff	34	52	36	78

0	0	0	1
1	0	0	0
0	1	0	0
0	0	1	0

1. What kind of assignment problems are there in school?
2. How do we find the solution?

Assignment Problems for School

Options available

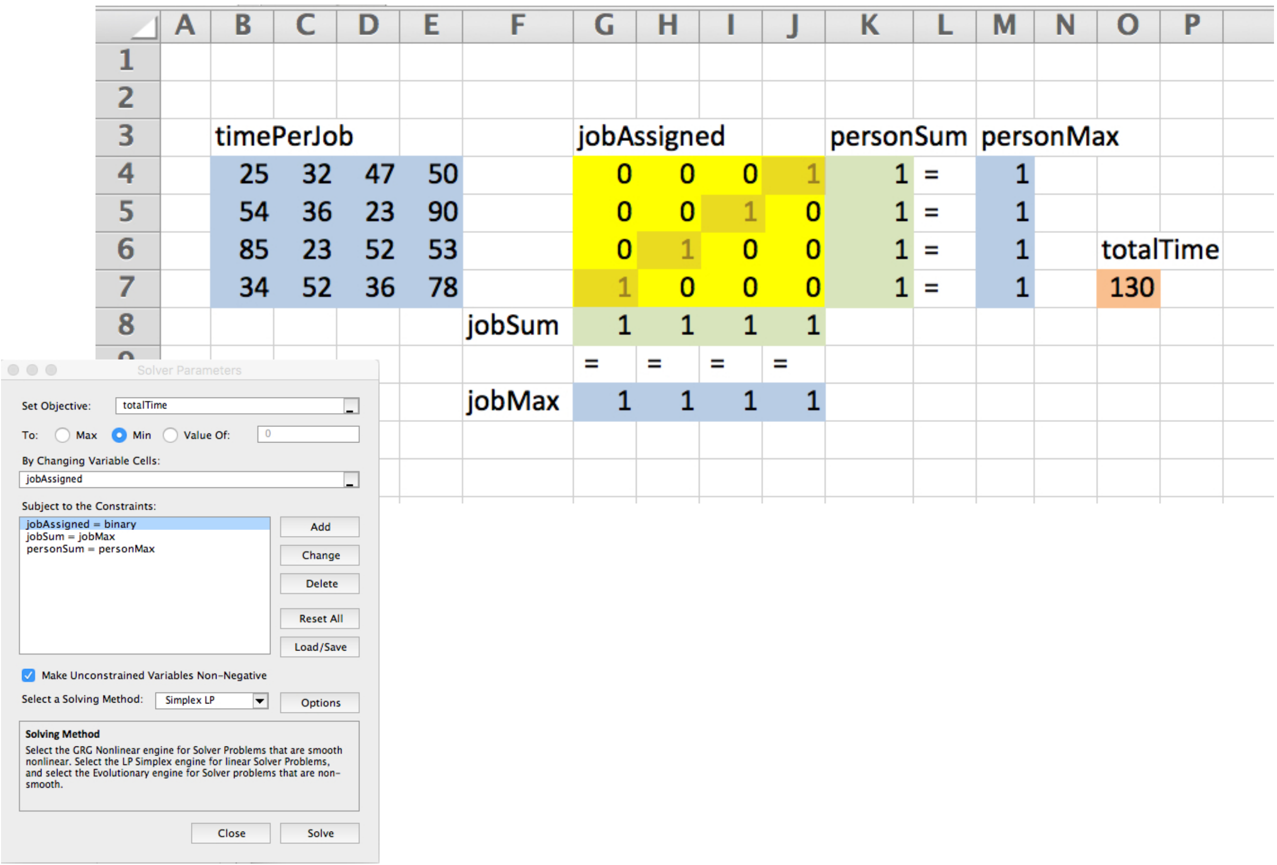
Pupils

Organising options: special weeks, sport choices, TecDays...

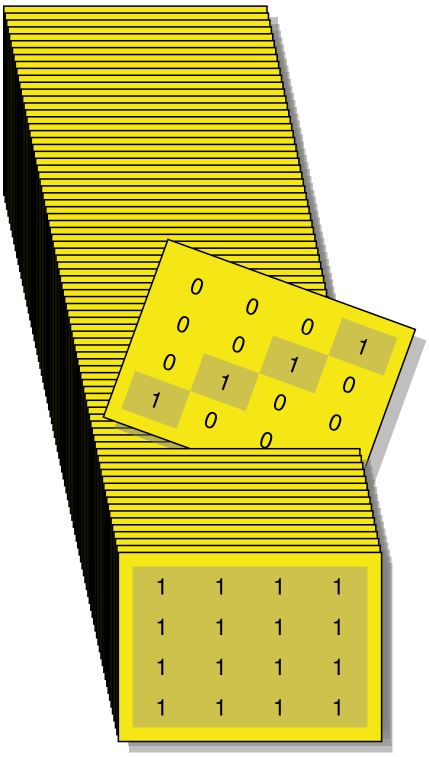
Maximise satisfaction

Class sizes, option availability

Spreadsheet Model



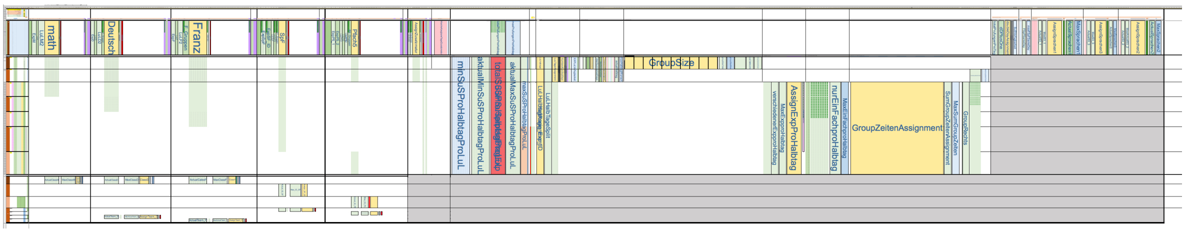
Brute Force



No	Objective	Optimal
Yes	174	174
Yes	202	174
Yes	158	158
Yes	142	142
Yes	243	142
Yes	130	130
Yes	190	130
Yes	173	130

Inefficiency

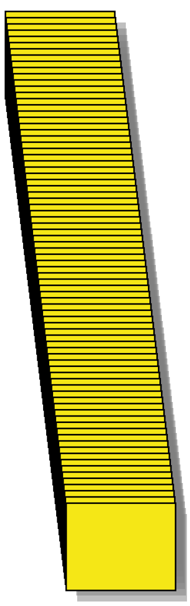
- Brute force is inefficient: $2^{\text{number of variables}}$ configurations!
- Our models are massive:

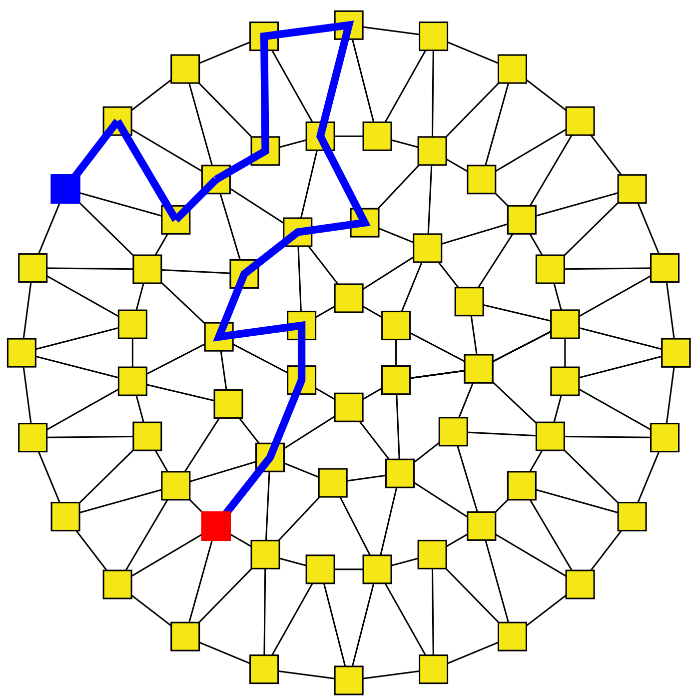


Here we have close to 50 000 variables.

- Enter: George Dantzig and his Simplex Algorithm! (1947)

Simplex Algorithm





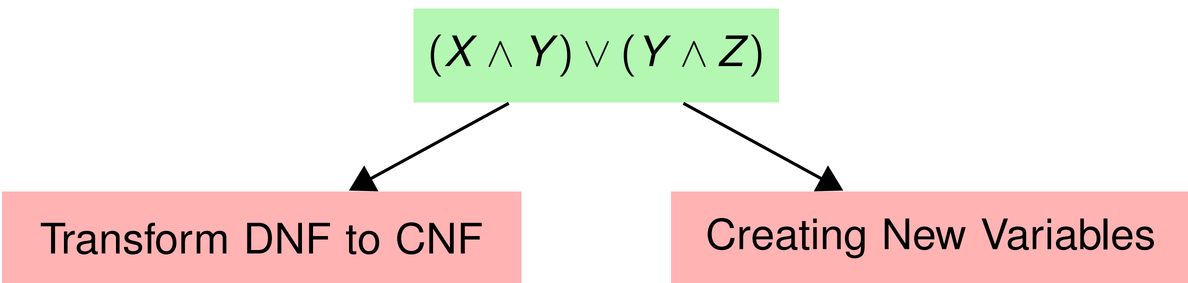
First find a solution, then optimise the objective.

Logical Connectives

	A	B	C	D	E	F
1		X	Y			
2						
3						
4		$X \vee Y$	$X + Y$	$\geq$	1	
5		$X \wedge Y$	$X + Y$	$\geq$	2	
6		$X \Rightarrow Y$	Y	$\geq$	X	
7						
8						

More Complex Formulas

	A	B	C	D	E
1		X	Y	Z	
2					
3					



Transforming DNF to CNF

$$DNF = (X \wedge Y) \vee (Y \wedge Z) \equiv (X \vee Y) \wedge (X \vee Z) \wedge (Y \vee Y) \wedge (Y \vee Z) = CNF$$

	A	B	C	D	E	F	G	H	I	J	K
1								X	Y	Z	
2											
3											
4								$X+Y$	$\geq$	1	
5								$X+Z$	$\geq$	1	
6								$Y+Y$	$\geq$	1	
7								$Y+Z$	$\geq$	1	
8											

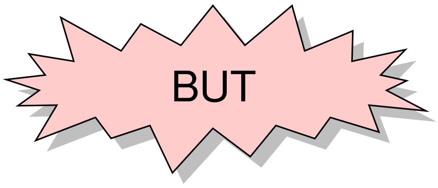
Creating New Variables

$$(X \wedge Y) \vee (Y \wedge Z)$$

	A	B	C	D	E	F	G	H	I	J	K
1		U	V					X	Y	Z	
2				$U+V$	$=$	1					
3											
4								$X+Y$	$\geq$	$2U$	
5								$Y+Z$	$\geq$	$2V$	
6											
7											
8											

Linearising the Non Linear

Simplex works when everything is linear.



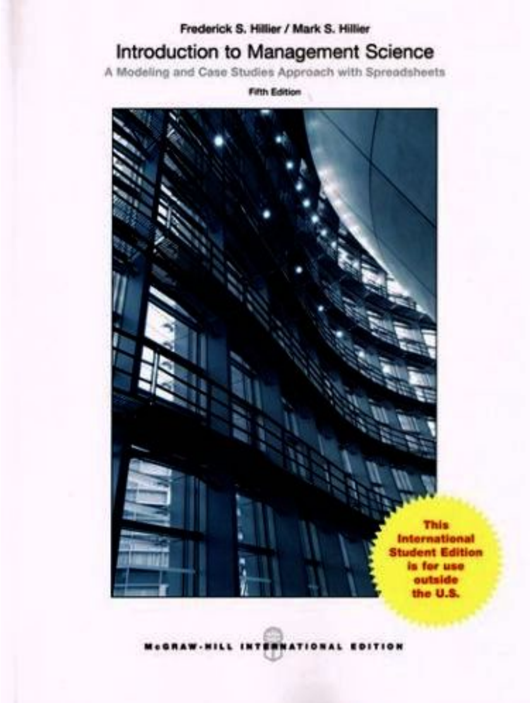
- Non linear objective function.
- Assigning simultaneous choices.
- Finding the minimum or maximum.
- Using the modulo function.

Finding the Maximum

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1																					
2		1	2	3	4		1	2	3	4											
3																					
4		0	1	1	1		0	0	0	1	1	$=$	1		15	$\leq$	16	$\leq$	28		4
5		1	1	0	0		0	1	0	0	1	$=$	1		4	$\leq$	4	$\leq$	6		2
6		1	0	0	0		1	0	0	0	1	$=$	1		2	$\leq$	2	$\leq$	2		1
7		0	1	1	0		0	0	1	0	1	$=$	1		7	$\leq$	8	$\leq$	12		3
8																					
9		1	2	4	8		2	4	8	16											
10																					

$$2^{Max-1} < \left(\sum_{i=0}^{Max-1} 2^i\right) + 1 \leq 2^{Max} \leq 2 \cdot \left(\sum_{i=0}^{Max-1} 2^i\right) < 2^{Max+1}$$

Acknowledgements



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