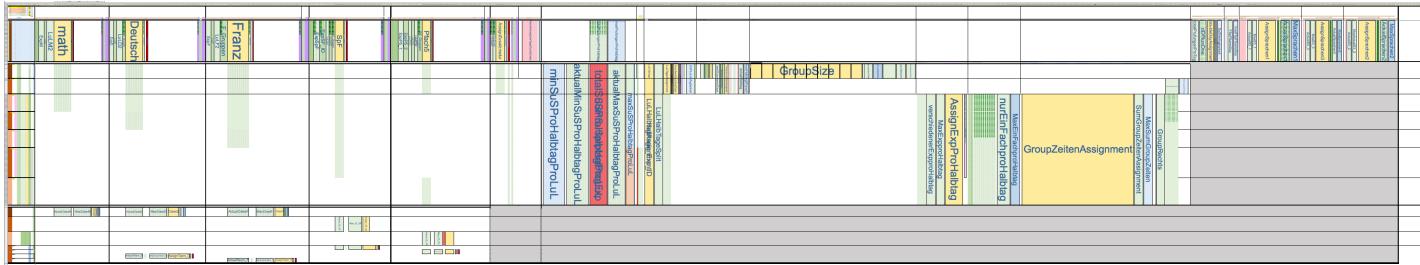


## Constraint Satisfaction Problems:

From the front of the class, to the back of the school.

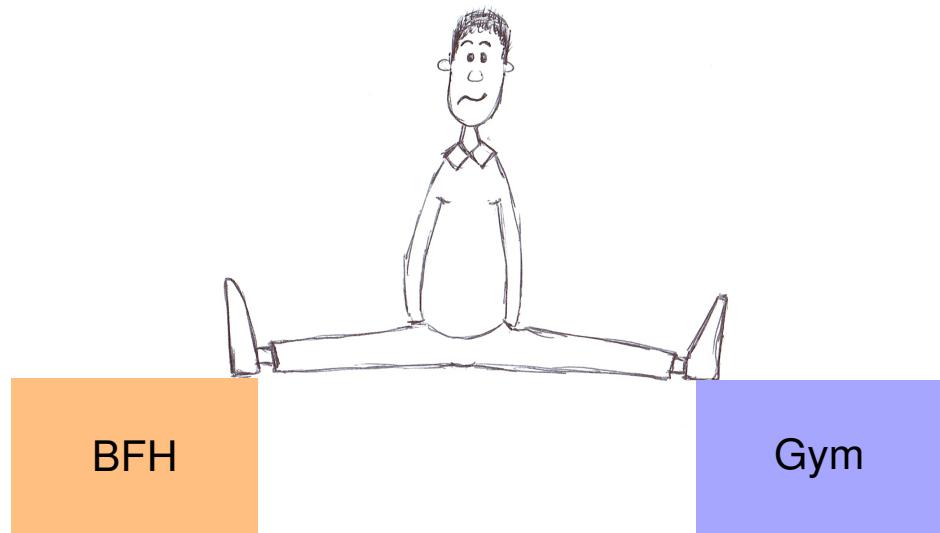


Geoff Ostrin

Berner Fachhochschule & Gymnasium Thun

Bern, 26.10.2019

# What's the Story?



# Serendipity



Constraint Satisfaction Problems:

From the front of the class, to the back of the school.

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## Constraint Satisfaction Problems

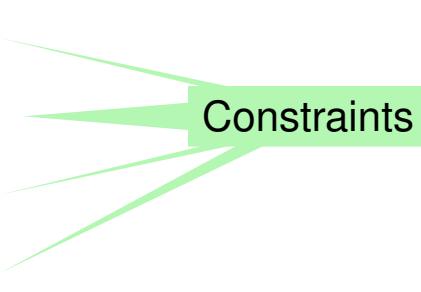
Find

$X$  and  $Y$   Decision Variables

such that we maximise

$Z = 300X + 500Y$   Objective Function

while satisfying

$$\begin{aligned} X &\leq 4 \\ 3X + 2Y &\leq 18 \\ X &\geq 0 \\ Y &\geq 0 \end{aligned}$$


Constraints

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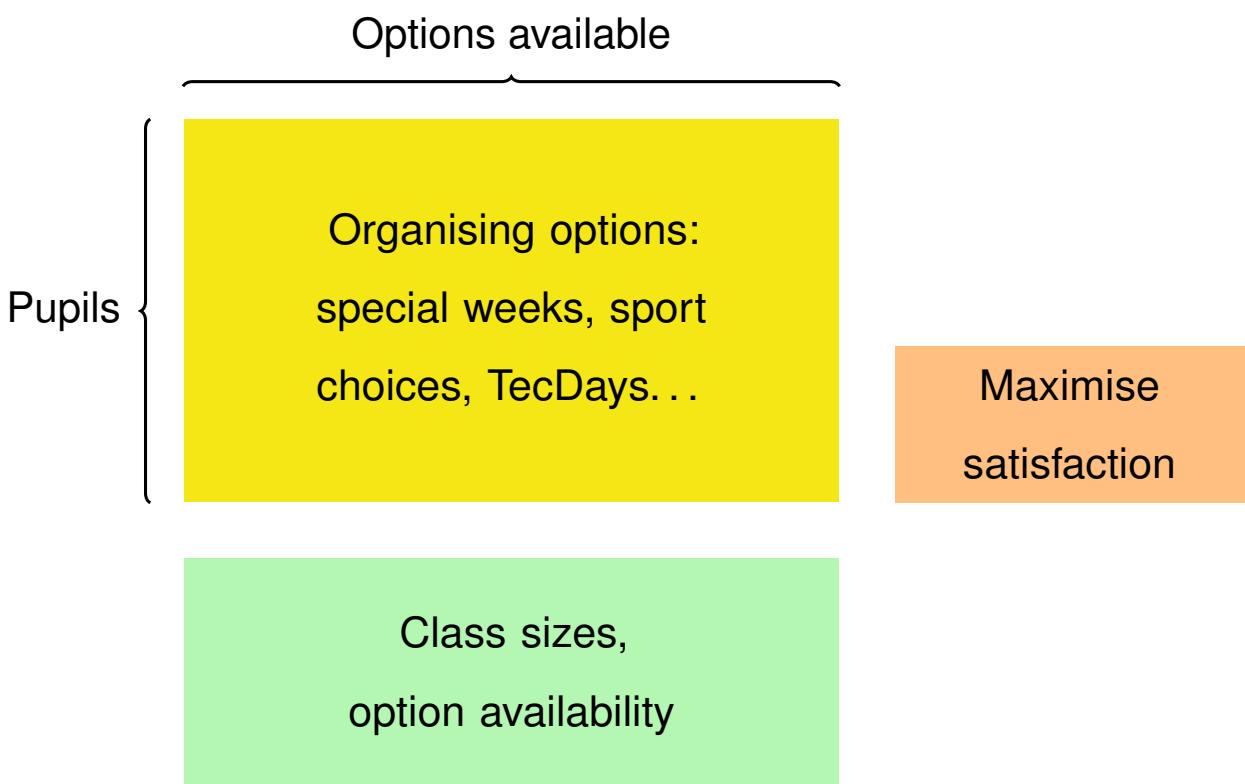
# Assignment Problems

	Email	Organise	Print	Arrange	
Leonard	25	32	47	50	0 0 0 1
Michel	54	36	23	90	1 0 0 0
Matthias	85	23	52	53	0 1 0 0
Geoff	34	52	36	78	0 0 1 0

1. What kind of assignment problems are there in school?
2. How do we find the solution?

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# Assignment Problems for School



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# Spreadsheet Model

The screenshot shows an Excel spreadsheet with the following data:

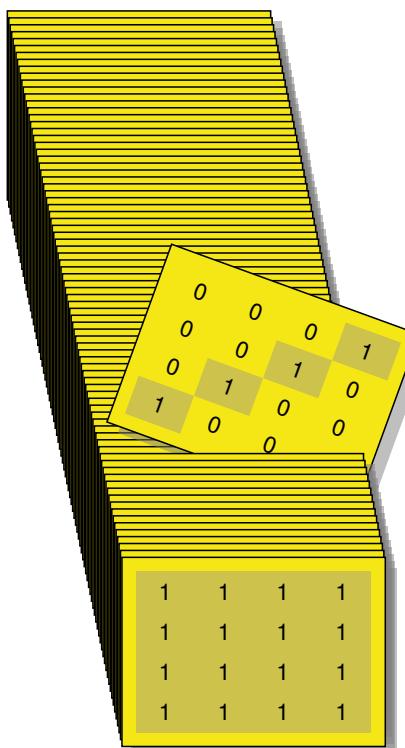
	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P
1																
2																
3		timePerJob					jobAssigned				personSum	personMax				
4		25	32	47	50		0	0	0	1	1	=	1			
5		54	36	23	90		0	0	1	0	1	=	1			
6		85	23	52	53		0	1	0	0	1	=	1		totalTime	
7		34	52	36	78		1	0	0	0	1	=	1		130	
8							jobSum	1	1	1	1					
								=	=	=	=					
							jobMax	1	1	1	1					

**Solver Parameters**

- Set Objective: totalTime
- To: Min
- By Changing Variable Cells: jobAssigned
- Subject to the Constraints:
  - jobAssigned = binary
  - jobSum = jobMax
  - personSum = personMax
- Make Unconstrained Variables Non-Negative
- Select a Solving Method: Simplex LP
- Solving Method: Select the GRG Nonlinear engine for Solver Problems that are smooth nonlinear. Select the LP Simplex engine for linear Solver Problems, and select the Evolutionary engine for Solver problems that are non-smooth.

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# Brute Force



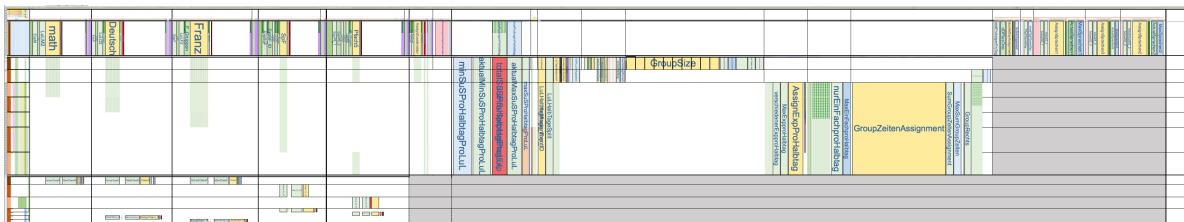
No

	Objective	Optimal
Yes	174	174
Yes	202	174
Yes	158	158
Yes	142	142
Yes	243	142
Yes	130	130
Yes	190	130
Yes	173	130

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# Inefficiency

- ▶ Brute force is inefficient:  $2^{\text{number of variables}}$  configurations!
  - ▶ Our models are massive:

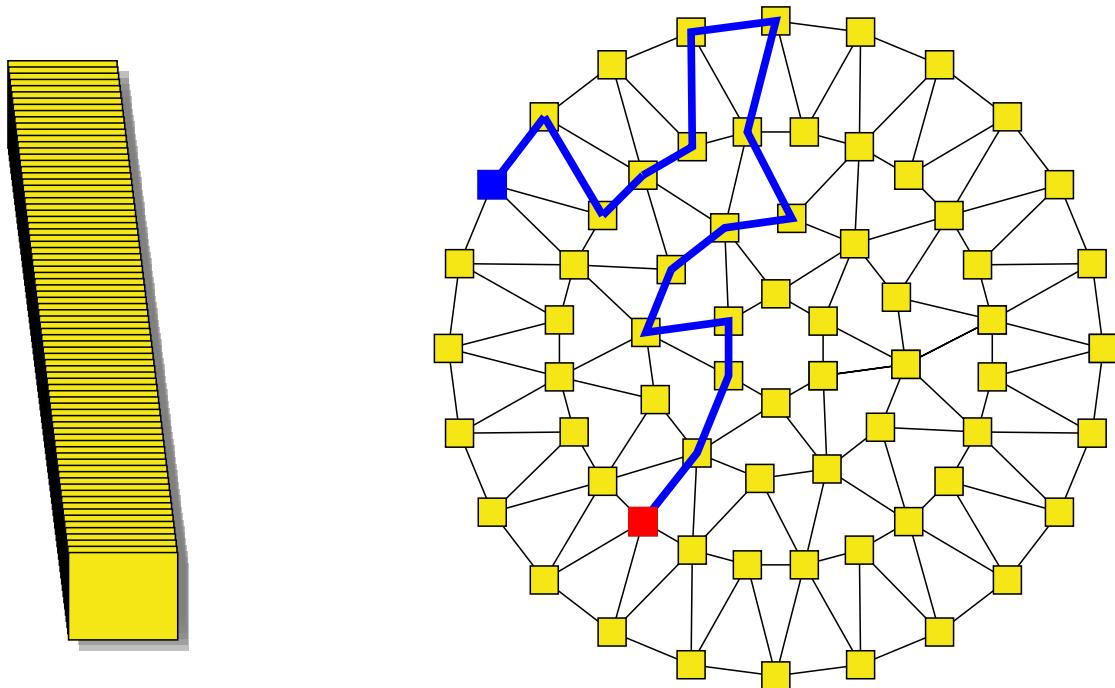


Here we have close to 50 000 variables.

- ▶ Enter: George Dantzig and his Simplex Algorithm! (1947)

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# Simplex Algorithm



First find a solution, then optimise the objective.

# Logical Connectives

	A	B	C	D	E	F
1		X	Y			
2						
3						
4		$X \vee Y$	$X + Y$	$\geq$	1	
5		$X \wedge Y$	$X + Y$	$\geq$	2	
6		$X \Rightarrow Y$	$Y$	$\geq$	$X$	
7						
8						

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# More Complex Formulas

	A	B	C	D	E
1		X	Y	Z	
2					
3					

$$(X \wedge Y) \vee (Y \wedge Z)$$

Transform DNF to CNF

Creating New Variables

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## Transforming DNF to CNF

$$DNF = (X \wedge Y) \vee (Y \wedge Z) \equiv (X \vee Y) \wedge (X \vee Z) \wedge (Y \vee Y) \wedge (Y \vee Z) = CNF$$

	A	B	C	D	E	F	G	H	I	J	K
1								X	Y	Z	
2											
3											
4								X + Y	$\geq$	1	
5								X + Z	$\geq$	1	
6								Y + Y	$\geq$	1	
7								Y + Z	$\geq$	1	
8											

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## Creating New Variables

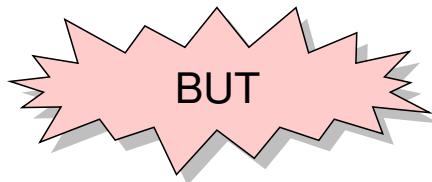
$$(X \wedge Y) \vee (Y \wedge Z)$$

	A	B	C	D	E	F	G	H	I	J	K
1		U	V					X	Y	Z	
2					U + V	=	1				
3											
4								X + Y	$\geq$	2U	
5								Y + Z	$\geq$	2V	
6											
7											
8											

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# Linearising the Non Linear

Simplex works when everything is linear.



- ▶ Non linear objective function.
- ▶ Assigning simultaneous choices.
- ▶ Finding the minimum or maximum.
- ▶ Using the modulo function.

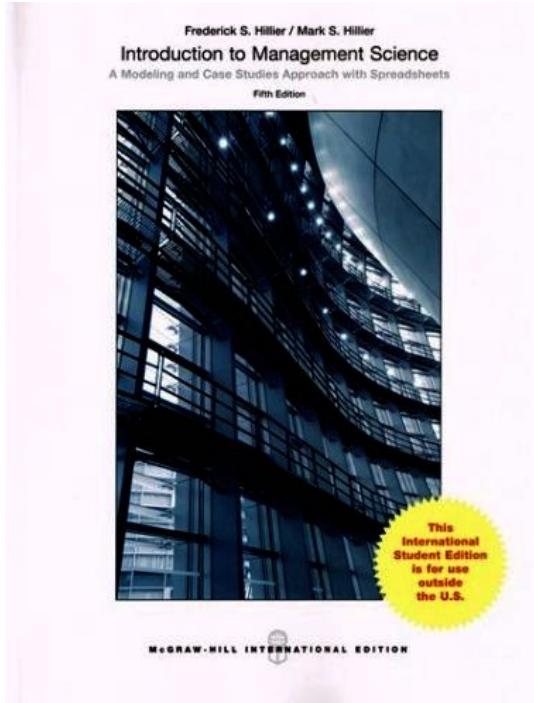
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## Finding the Maximum

	A	B	C	D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U
1																					
2		1	2	3	4		1	2	3	4											
3																					
4		0	1	1	1		0	0	0	1	1	=	1		15	$\leq$	16	$\leq$	28	4	
5		1	1	0	0		0	1	0	0	1	=	1		4	$\leq$	4	$\leq$	6	2	
6		1	0	0	0		1	0	0	0	1	=	1		2	$\leq$	2	$\leq$	2	1	
7		0	1	1	0		0	0	1	0	1	=	1		7	$\leq$	8	$\leq$	12	3	
8																					
9		1	2	4	8		2	4	8	16											
10																					

$$2^{Max-1} < \left( \sum_{i=0}^{Max-1} 2^i \right) + 1 \leq 2^{Max} \leq 2 \cdot \left( \sum_{i=0}^{Max-1} 2^i \right) < 2^{Max+1}$$

# Acknowledgements



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- ▶ Leonard Kwuida

[geoffrey.ostrin@bfh.ch](mailto:geoffrey.ostrin@bfh.ch)

[www.schullogistik.ch](http://www.schullogistik.ch)